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In this paper we discuss the quantum coherence properties of the FEL within the single electron non-relativistic Hamiltonian picture. We analyze the problem both in the single and multimode hypothesis.

QUANTUM COHERENCE PROPERTIES OF THE FEL

G.Dattoli, J.Gallardo, A.Renieri, M.Richetta, A.Torre

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QUANTUM COHERENCE PROPERTIES OF THE FEL

G. Dattoli, J.C.Gallardo(+), A. Renieri, M. Richetta(++), and A. Torre(+++)

ENEA, Dip. TIB - Divisione Fisica Applicata,
Centro Ricerche Energia Frascati,
C.P. 65 - 00044 Frascati, Rome, Italy.

Abstract

In this paper we discuss the quantum coherence properties of the FEL within the framework of the single electron non-relativistic Hamiltonian picture. We analyse the problem both in the single and multi-mode hypotheses.

(+) Quantum Institute, University of California at Santa Barbara

(++) ENEA Student

(+++) I.N.F.N. Sez. Napoli

The quantum non-relativistic Hamiltonian picture [1] of the Free Electron Laser (FEL) has provided a useful tool to understand the coherence (*) properties of this new kind of laser device [2-5].

In this paper we will review the main features of that analysis, relevant to a single mode - single electron picture. Furthermore we show how the longitudinal multimode effects can be properly accounted for, in a quantum mechanical way.

The basic elements of the non-relativistic-quantum Hamiltonian Theory of the FEL will not be reported here, but they can be found in many review articles (as e.g. Ref. [6] where a detailed account of both classical and quantum FEL treatments has been given).

The non-relativistic single mode - single electron FEL Hamiltonian reads

$$H = \frac{p^2}{2m} + \hbar\omega(a_L^+ a_L + 1/2) + \hbar\omega(a_U^+ a_U + 1/2)$$

$$+ \hbar\Omega[a_L^+ a_U e^{-2ikz} + a_U^+ a_L e^{2ikz}] \tag{1}$$

(For the symbols see Table I)

The physical meaning of (1) is transparent. The first three terms in (1) account for the electron energy and free field ones, while the FEL mechanism arises as due to the fourth term in the Hamiltonian, which causes:

(*) From now on when we say coherence we refer to quantum coherence and not to the spatial and temporal ones.

a) The creation of a laser photon, the destruction of an undulator one and the loss of $2\hbar K$ of electron momentum.

b) The conjugate process, namely the destruction of a laser photon, the creation of an undulator one and the gain of $2\hbar K$ of electron momentum.

It is well-known that [6] the Hamiltonian (1) allows two laws of conservation relevant to the total number of photons (laser + undulator) and to the total momentum (electron + fields).

The above denoted conservation laws suggest that the evolution of the FEL operation states can be characterized by a single integer only, namely the number of exchanged photons during the interaction.

In other words, if we start from a single state with electron energy $P_0^2/2m$ and $n^0_{L,U}$ initial photon number, the following states only are coupled

$$|\psi\rangle = \exp\left\{-i\left[\frac{P_0^2}{2m\hbar} + \omega(n_L^0 + n_U^0 + 1)\right]t\right\} \cdot \cdot \sum_{\ell=0}^{\infty} C_\ell(t)\,|P_0 - 2\ell\hbar k,\ \ n_L^0 + \ell,\ \ n_U^0 - \ell\rangle \tag{2}$$

where ℓ is the number of exchanged photons and $C_\ell(t)$ are time dependent coefficients.

If we assume that the initial "laser" field is the vacuum ($n_L^0 = 0$), we find that the coefficients C_ℓ obey the following differential-difference equation

$$i \frac{dC_\ell}{d\tau} = (-w_0 + \varepsilon\ell)\ell C_\ell + \bar{\Omega}[\sqrt{(\ell+1)(n_U^0-\ell)}\; C_{\ell+1} +$$

$$+ \sqrt{\ell(n_U^0 - \ell + 1)}\; C_{\ell-1}], \qquad C_\ell(0) = \delta_{\ell,0} \tag{3}$$

The above expression is known as the Spherical or SU_2 - Raman-Nath (R.N.) equation and has been recently discussed in Refs [7]. It cannot be solved exactly by means of known functions, but a perturbed analysis in terms of the parameter ε, linked to the quantum electron recoil (see Table I), can be accomplished. Let us notice that at the zeroth order in ε, and thus in the quantum corrections, the solution of (1) can be written quite straightforwardly [7], so that the probability of emitting ℓ photons in the recoilless approximation is given by

$$|C_\ell^0|^2 = \binom{n_U^0}{\ell} P^\ell (1 - P)^{(n_U^0-\ell)} \tag{4}$$

where

$$P = \bar{\Omega}^2 \left(\frac{\sin(\delta\tau/2)}{\delta/2}\right)$$

$$\delta = \sqrt{w_0^2 + 4} \tag{5}$$

It is easy to recognize (4) as a binomial distribution. However, since the number of undulator "photons" is always a large quantity, taking the limit $n_U^0 >> \ell$ (4) gives

$$|C_\ell|^2_{\varepsilon=0,\, n_U^0>>\ell} \simeq \frac{1}{\ell!}\, |\alpha(\tau)|^{2\ell} e^{-|\alpha(\tau)|^2}$$

$$\alpha(\tau) = (-i)\exp\left\{ i w_0 \tau/2 \right\} \Omega_R \frac{\sin(w_0\tau/2)}{w_0/2} \tag{6}$$

Needless to say, the above expression (6) is a Poisson distribution, therefore up to the zeroth order in the quantum corrections we have recovered a result strongly reminiscent of the conventional laser statistics.

We could also show, in a more rigorous way, within the framework of the same approximation, that the states (2) evolve from the vacuum into coherent Glauber states $^{(*)}$.

It is easy to realize that the recoilless approximation amounts to a gainless process [4,8]. The inclusion of ε is, therefore, necessary if we want to find meaningful results. If we look only for first order quantum corrections we derive, however, significant deviation from the previously recovered results [2,4,8]. Namely

a) We cannot prove that the states (2) evolve from the vacuum into Glauber ones.

b) The probability of emitting ℓ photons is no longer a Poisson situation but is given by

$$|C|^2_{\varepsilon\neq 0,\, n_U^0>>\ell} = e^{-|\alpha(\tau)|^2} \frac{|\alpha(\tau)|^{2\ell}}{\ell!} \cdot$$

$$\cdot \left\{ 1 + \frac{2\varepsilon}{|\alpha(\tau)|} \frac{\partial}{\partial w_0} |\alpha(\tau)| \cdot [(2\ell+1)|\alpha(\tau)|^2 - \alpha(\tau)|^4 - \ell^2] + \ldots \right\} \tag{7}$$

$^{(*)}$ For a more detailed account see Refs [4].

c) Furthermore we can expect genuine quantum effects such as antibunching and squeezing.

Evaluating, indeed, the quantity

$$<\Delta \ell^2> - <\ell> \simeq - \varepsilon \frac{\partial}{\partial w_0} |\alpha(\tau)|^4 \tag{8}$$

We find that, in the FEL gainless process, the photon number fluctuations are just the square root of the number of the emitted photons, as in conventional , while, if there is gain or loss, bunching or antibunching arise. (See also Refs [2,4,8] for further comments.)

As to the squeezing, if we introduce the operators

$$A_1 = \frac{1}{2}(A^+ + A) \qquad (A^+ = a^+ e^{-2ikz})$$

$$A_2 = \frac{1}{2i}(A^+ - A) \tag{9}$$

which realize a kind of electron-field p,q variables we can evaluate

$$<\Delta A_1^2> \simeq \frac{1}{4} + \frac{1}{2}\varepsilon\left(\frac{\partial}{\partial w_0}|\alpha(\tau)|^2\right)\cos w_0\tau + \ldots$$

$$<\Delta A_2^2> \simeq \frac{1}{4} - \frac{1}{2}\varepsilon\left(\frac{\partial}{\partial w_0}|\alpha(\tau)|^2\right)\cos w_0\tau + \ldots \tag{10}$$

We must, therefore, underline that both antibunching and squeezing are related to the gain process and are the only genuine quantum effects which can be derived within this analysis.

The above results hold in the hypothesis that the initial laser field is the vacuum.

Let us discuss the case of an arbitrary coherent Glauber state as initial "laser" field. This is the physical situation of an amplification FEL experiment and we are going to discuss in which way the coherence properties of a conventional laser, undergoing amplification, may be affected by the interaction.

To carry out this analysis we need only a few modifications to the above procedure. Since we do not start from a state with a fixed number of photons, but from a coherent one we must rewrite (2) as(*)

$$|\psi\rangle = \exp\left\{-i\left[\frac{P_0^2}{2m\hbar} + \omega(n_u^0+1)\right]t\right\} \cdot$$

$$\cdot \sum_{n=0}^{\infty} \sum_{\ell=-n}^{\infty} \frac{e^{-|\alpha_0|^2/2}}{\sqrt{n!}} \alpha_0^n \, C_\ell^n(t) \, |n;\ P_0 - 2\ell\hbar k;\ \ell\rangle \tag{11}$$

In this case the differential equation for the C_ℓ coefficients writes

$$i\,\frac{dC_\ell^n}{d\tau} = (-w_0 + \varepsilon\ell)\ell C_\ell^n + \Omega_R\left[\sqrt{n+\ell+1}\; C_{\ell+1}^n + \right.$$

$$\left. + \sqrt{n+\ell}\; C_{\ell-1}^n\right], \qquad C_\ell^n(0) = \delta_{\ell,0} \tag{12}$$

The above expression is known as the Harmonic R.N. equation recently analysed in Refs [9]. Even in this

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(*) From now on we assume that the number of undulator "photons" be very large and that its variation can be neglected, further comment on this assumption will be given in the following.

case, an exact solution cannot be found in terms of known functions; but, as before, we will carry out only a first order analysis in terms of the electron recoil.

The zeroth order solution can be easily found (9) and writes

$$C_{\ell}^{n}\Big|_{\varepsilon=0} = \exp\left\{i\frac{w_0}{2}\int_0^{\tau}|\alpha(\tau')|^2 d\tau'\right\} \phi_n^{\ell}[\alpha(\tau)] \tag{13}$$

where

$$\phi_n^{\ell}[\alpha(\tau)] = \sqrt{\frac{n!}{(n+\ell)!}}\; e^{-|\alpha(\tau)|^2/2}(\alpha(\tau))^{\ell} L_n^{\ell}[|\alpha(\tau)|^2]$$

with $L_n^{\ell}(\cdot)$ generalized Laguerre polynomials.

We must now stress a significant difference with respect to the previous case. Even in the recoilless approximation we cannot prove that (11) is a Glauber state at any time.

This is due to the fact that the creation annihilation operators, acting on our Hilbert space, characterized by the integer ℓ only, affect both the fields and electron variables see (9),(8), so that(*)

$$A|\psi\rangle_{\varepsilon=0} = \left(\alpha(\tau) + \alpha_0 e^{2ikz}\right)|\psi\rangle_{\varepsilon=0} \tag{14}$$

The relationship (14) does not define a coherent state, because e^{2ikz} is not a c-function but an opera-

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(*) If A were a pure field operator we could easily prove

$A|\psi\rangle_{\varepsilon=0} = (\alpha(\tau) + \alpha_0)|\psi\rangle_{\varepsilon=0}$

and thus Glauber coherence

tor. Notwithstanding, we can still prove that, at the zeroth order in ε, we can recover the well-known results of the Poisson statistics. However, if we want to recover the gain we must look for a first order perturbed expansion in the recoil. Therefore, including the lowest order quantum corrections, the evolution of the states (11) reads

$$|\psi\rangle_{\varepsilon\neq 0} = \exp\left\{ i\frac{w_0}{2}\int_0^\tau |\alpha(\tau')|^2 d\tau' + i\omega\Delta t(n_U^0+1)\tau + \right.$$

$$\left. + i\frac{P_0^2}{2m\hbar}\Delta t\tau \right\} \cdot \sum_{n=0}^{\infty} e^{-|\alpha_0|^2/2}\alpha_0^n \sum_{\ell=-n}^{\infty} \sqrt{\frac{1}{(n+\ell)!}} \cdot [\alpha(\tau)]^{\ell} \cdot$$

$$\cdot\ e^{-|\alpha(\tau)|^2/2} \cdot [A_{\ell,n} + iD_{\ell,n}]\,|n_i P - 2\ell\hbar k \;;\ \ell\rangle \qquad (15)$$

where $A_{\ell,n}$ and $D_{\ell,n}$ are somewhat complicated expressions given by

$$A_{\ell,n} \simeq L_n^{\ell}(\cdot) + \frac{\varepsilon}{|\alpha(\tau)|}\frac{\partial\alpha(\tau)}{\partial w_0}\left\{(2\ell+1)L_n^{\ell+1}(\cdot)|\alpha(\tau)|^2 - \right.$$

$$\left. - (2\ell-1)(n+\ell)L_n^{\ell-1}(\cdot)\right\} - \frac{\varepsilon}{|\alpha(\tau)|}\frac{\partial|\alpha(\tau)|}{\partial w_0} \cdot$$

$$\cdot \left\{|\alpha(\tau)|^4 L_n^{\ell+2}(\cdot) - (n+\ell)(n+\ell-1)L_n^{\ell-2}(\cdot)\right\}$$

$$D_{\ell,n} \simeq \frac{\varepsilon\tau}{2}\left\{(2\ell+1)|\alpha(\tau)|^2 L_n^{\ell+1}(\cdot) + (2\ell-1)(n+\ell)L_n^{\ell-1}(\cdot)\right\} -$$

$$- \frac{\varepsilon\tau}{2}\left\{|\alpha(\tau)|^4 L_n^{\ell+2}(\cdot) + (n+\ell)(n+\ell-1)L_n^{\ell-2}(\cdot)\right\} \qquad (16)$$

The above quantities can be exploited to evaluate a number of interesting quantities such as the average photon number, the second moment of the distribution and so on. The explicit calculation of these quantities is tedious but straightforward it requires a repeated application of the well-known properties of the Laguerre polynomials [10]; so that after some algebra we find

$$\langle n+\ell \rangle_{\varepsilon \neq 0} \simeq |\alpha_0|^2 + |\alpha(\tau)|^2 - \varepsilon \frac{\partial |\alpha(\tau)|^2}{\partial w_0} [1 + 2|\alpha_0|^2] \quad . \tag{17}$$

It is easy to recognize the physical meaning of the various terms appearing in (17). The first term is the average number of photons of the input field, the second one accounts for the spontaneously emitted photons and finally the third term is the stimulated part (the extra term 1 comes from the vacuum field fluctuations).

We can also evaluate the quantity

$$\langle \Delta(n+\ell)^2 \rangle_{\varepsilon \neq 0} - \langle n+\ell \rangle_{\varepsilon \neq 0} \simeq -\varepsilon (2|\alpha_0|^2+1) \frac{\partial |\alpha|^4}{\partial w_0} - 2\varepsilon |\alpha_0|^2 \frac{\partial |\alpha|^2}{\partial w_0} \tag{18}$$

Thus finding again that we can have bunching or anti-bunching according to the case $w_0 \gtrless 0$ (see Fig. 1). Similar conclusions can be drawn for the squeezing.

Before discussing the problem of the multimode analysis we need some comments on the assumption of constant undulator photon numbers during the interaction. It is well-known that the concept of undulator photon is grounded on the Weizsäcker-Williams approximation [6]. Furthermore it is also well-known that an undulator with a static field of 1 kG has an equivalent photon number density of about $10^{24}/cm^3$, which is so large that it is practically unaffected by the interaction. Our assumption is therefore well justified. Notwithstanding it is not correct for a proper and rigorous analysis of the FEL coherence. We should indeed treat the undulator field as a coherent Glauber one, in this way we reduce the problem of FEL coherence to that of the evolution of two coupled coherent fields. This problem may be understood in terms of the evolution of the so-called coherent Bloch states [12]. In this case even when we start from a zero laser field we meet with the problem of the electron-field operators, discussed before. Therefore, strictly speaking, the problem of the FEL Glauber coherence even at the zeroth order in ε is still open.

The main implicit assumption of the up to now developed analysis is that the electron has a fixed initial momentum and that it is completely delocalized. We will now assume that the initial electron wave-function is an arbitrary packet and that the laser field may be expanded into many longitudinal modes [12].(*) Therefore we have that the Hamiltonian now writes

$$H = \frac{p^2}{2m} + \hbar\omega_U(a_U^+a_U+1/2) + \hbar \sum_{s=1}^{n} \omega_s(a_s^+a_s+1/2) \quad +$$

(*)A proper analysis of the FEL signal starting from the vacuum requires a multimode expansion even in the hypothesis of continuous electron beam.

$$+ \hbar \sum_{s=1}^{n} \Omega_{s,w} \left\{ a_u^+ a_s \exp[i(k_s + k_U)z] + h.c. \right\} +$$

$$+ \hbar \sum_{\substack{s,j \\ j<s}} \Omega_{s,j} \left\{ a_s^+ a_j \exp[i(k_s - k_j)z] + h.c. \right\} . \tag{19}$$

The physical meaning of (19) is self-explanatory.

We have a further term namely the fourth one which accounts for the laser-laser interaction (for further comments see Ref. [12]).

According to the Hamiltonian (19), to the assumption of arbitrary wave packet for the electron and to the laws of conservation(*) we can write the analogous of the states (2) as

$$|\psi(\tau)\rangle = \sum_{\{\ell\}} \int dk_e g(k_e) C_{\{\ell\}}(k_e,\tau) \cdot$$
$$\cdot \exp\left\{ - i\hbar \frac{k_e^2}{2m} \Delta t + n_U^0 \omega_U \Delta t + \sum_{j=1}^{n} n_j^0 \omega_j \Delta t] \tau \right\} \cdot$$
$$|k_e - \sum_{s=1}^{n} (k_s + k_u)\ell_s, \{n_s^0 + \ell_s\}, n_U^0 - \sum_{s=1}^{n} \ell_s\rangle \tag{20}$$

It is also easy to prove that the coefficient $C_{\{\ell\}}(k_e, \tau)$ may be determined from the following equations

$$i \frac{dC_{\{\ell\}}(k_e,\tau)}{d\tau} = \sum_{j=1}^{n} [- \eta_j + \sum_s \varepsilon_{s,j} \ell_s] \ell_j C_{\{\ell\}}(k_e,\tau) +$$

$$+ \sum_{j=1}^{n} (\bar{\Omega}_{j,U}) \Big\} \sqrt{(n_j^0 + \ell_j + 1)(n_U^0 - \sum_q \lambda_q)}\, C_{\{\ell_{j+1}\}} +$$

(*)The laws of conservation are again relevant to the total momentum and total number of photons. See Ref. [12] for the specific details.

$$+ \sqrt{(n_j^0 + \ell_j)(n_U^0 - \sum_q \ell_q + 1)}\; C_{\{\ell_j - 1\}} \quad +$$

$$+ \sum_{\substack{s,j \\ s<j}} \bar{\Omega}_{sj} \Big\{ \sqrt{(n_s^0 + \ell_s)(n_j^0 + \ell_j + 1)}\; C_{\{\ell_s - 1;\, \ell_j + 1\}} \quad +$$

$$+ \sqrt{(n_s^0 + \ell_s + 1)(n_j^0 + \ell_j)}\, C_{\{\ell_s + 1;\, \ell_j - 1\}} \Big\}; \quad C_{\{\ell\}}(0) = \prod_{s=0}^{n} \delta_{\ell_s, 0} \tag{20}$$

The above equation is an example of SU_n-R.N. [14] equation we do not dwell (20) on its general form but we make some simplificative assumptions.

(1) We neglect the undulator photon number variation and start from the vacuum.

(2) We neglect the laser-laser interaction.

So that (20) reduces to

$$i \frac{dC_{\{\ell\}}}{d\tau} = \sum_{j=1}^{n} \Big[-\eta_j + \sum_s \varepsilon_{s,j} \ell_s \Big] \ell_j\, C_{\{\ell\}} \quad +$$

$$+ \sum_{j=1}^{n} \bar{\Omega}_{Rj} \Big\{ \sqrt{\ell_j + 1}\; C_{\{\ell_j + 1\}} \quad +$$

$$+ \sqrt{\ell_j}\; C_{\{\ell_j - 1\}} \Big\},$$

$$C_{\{\ell\}}(0) = \prod_{s=1}^{n} \delta_{\ell_s, 0} \tag{21}$$

If we neglect the electron recoil the equation (21) is solved straightforwardly and reads

$$C_{\{\ell\}}(k_e,\tau)\Big|_{\varepsilon_{s,j}=0} = \prod_{j=1}^{n} \frac{1}{\sqrt{\ell_j}} \exp\left\{ i\,\frac{\eta_i}{2}\int_0^{\tau} |\alpha_j(\tau')|^2 d\tau' \right\} \cdot$$

$$\cdot \exp\left\{-\frac{1}{2}|\alpha_j(\tau)|^2\right\} \cdot (\alpha_j(\tau))^{\ell_j}$$

$$\alpha_j(\tau) = (-i)\Omega_{Rj} \exp\left\{ i\eta_j \frac{\tau}{2}\right\} \left(\frac{\sin(\eta_j \tau/2}{\eta_j/2} \right) \tag{22}$$

Therefore we find that the probability of finding ℓ_j photons at the time τ disperses as a Poissonian. The inclusion of the electron recoil, even at the first order, is a quite complicated job [13], but just to give an idea we have evaluated the average number of photons in the n-th mode, including the first order quantum correction, thus finding

$$\begin{aligned}
\langle \ell_m \rangle = \int_{-\infty}^{+\infty} |g(k_e)|^2 \Big\{ & |\alpha_m(\tau)|^2 - \\
& - \varepsilon_{mm} \frac{\partial}{\partial \eta_m} |\alpha_m(\tau)|^2 + 2 \sum_j \varepsilon_{jm} [\, \mathbb{R}_e R(\tau, \eta_j, \eta_m) - \\
& - \frac{2}{3} |\alpha_j(\tau)|^2 \frac{\partial}{\partial \eta_m} |\alpha_m(\tau)|^2] |\alpha_m(\tau)| + \\
& + 2 \sum_j \varepsilon_{j,m} \left(\frac{\partial}{\partial \eta_j} |\alpha_j(\tau)|^2 \right) |\alpha_m(\tau)|^2 - \\
& - 2 \sum_j \varepsilon_{j,m} |\alpha_j(\tau)|^2 \frac{\partial}{\partial \eta_m} |\alpha_m(\tau)|^2 + 3\Omega_{Rm} \tau \cdot \\
& \cdot \frac{\sin \eta_m \tau}{\eta_m^3} \sum_j \varepsilon_{jm} \frac{(\eta_m^2 - \eta_j^2)}{\eta_j} |\alpha_j(\tau)|^2 \Big\} dk_e .
\end{aligned} \tag{23}$$

where $R(\tau, \eta_i, \eta_m)$ is a somewhat complicated function and it will be reported elsewhere [13].. From (23) it follows that in first order in $\Omega^2_{R_j}$ the laser modes evolves independently of each other but becomes coupled for higher order of the coupling parameter. This result should be campared to that of Ref. [12] where it was shown that the gain of the FEL in the longitudinal multimode hypothesis is linked to the Fourier-transform of the longitudinal electron-beam distribution. Finally, we want to point out that the above outlined multimode quantum theory may be the basis for the understanding of the growth of the FEL signal from the noise [13].

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Table I List of the symbols used throughout the text

m	Electron mass
c	Light velocity
$\hbar$	Planck constant
$r_0 = e^2/mc^2$	Classical electron radius
P^0, z	Electron longitudinal momentun and coordinate
$b_{L,U}$, $b^+_{L,U}$	Annihilation and creation operators for laser and undulator
$\vec{k} = -\vec{k}_U$	Laser and undulator wave-vectors in the moving frame
$\omega_0 = \lvert k \rvert \cdot c$	Laser (undulator) frequency
$n_{L,U}$	Number of photons of the laser and the undulator
Δt	Interaction time
V	Interaction volume
w_0	$2\omega\ \Delta t\ P_0/mc$
ε	$2\hbar\ k^2/m\ \Delta t$
Ω	$2\pi c^2 \Omega_0/\omega V$
$\bar{\Omega}$	$\Omega \Delta t$
Ω_R	$\bar{\Omega}\ \sqrt{n_U^0}$
$\alpha_0 = \lvert \alpha_0 \rvert e^{-i\omega t}$	$\lvert \alpha_0 \rvert \equiv$ average number of photons in the input laser field
k_s	s-th laser mode wave vector
k_e	electron wave vector

$g(k_e)$	electron wave packet distribution in the momentum representation
η_i	$[(\omega_U-\omega_i)+(\Omega_U-\Omega_i)+\frac{\hbar k_e}{m}(k_i+k_U)]\Delta t$
Ω_{Ri}	$\frac{2\pi c^2 r_0}{(\omega_i\omega_U)^{\frac{1}{2}}}\sqrt{n_U^0}\,\Delta t$
ε_{sj}	$\frac{\hbar}{2m}(k_s+k_U)(k_j+k_U)\Delta t.$